Effective Field Theory for Nuclear Physics

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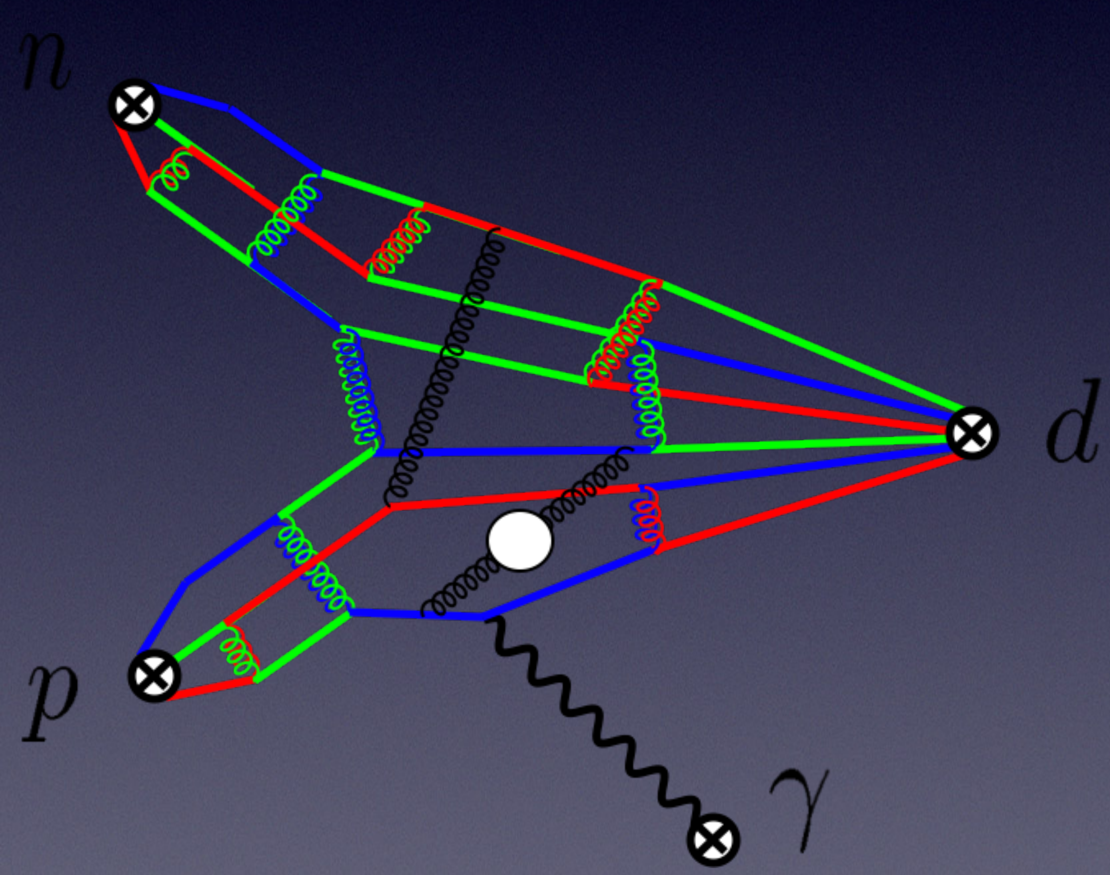
Overview

- Introduction
- Basic ideas of EFT
- Basic Examples of EFT
- Algorithm of EFT
- Review NN scattering
- NN scattering in Effective Field Theory

- Radiative neutron capture in C-14
- Form factors of one nucleon halo C-15

Introduction

- Basic Building blocks of Nature - Quarks and Leptons
- QCD and QED describes the dynamics of fundamental particles
- Describe Low Energy Physics(Light Nuclei) with QCD is complicated,non perturbative, and infinite body problem.
- Possible Solution
    - Lattice QCD
    - EFT



Basic ideas of EFT

- Dynamics at long distance do not depend on what happens at short distance
- Low energy interactions do not care about details of high energy interactions
- We don't need to understand Nuclear Physics to build a bridge !!

EFT Concepts

- EFT is a systematic approximation to some underlying dynamics, valid in specific regime
- Phenomena at low energy (long wave length) can not probe detail of high energy (short distance) structure of Physics
- Expand short distance physics in terms of contact interaction
- Coefficients fit to low energy data

Some Basic Examples

Gravity for h < R

- Gravitational potential energy,

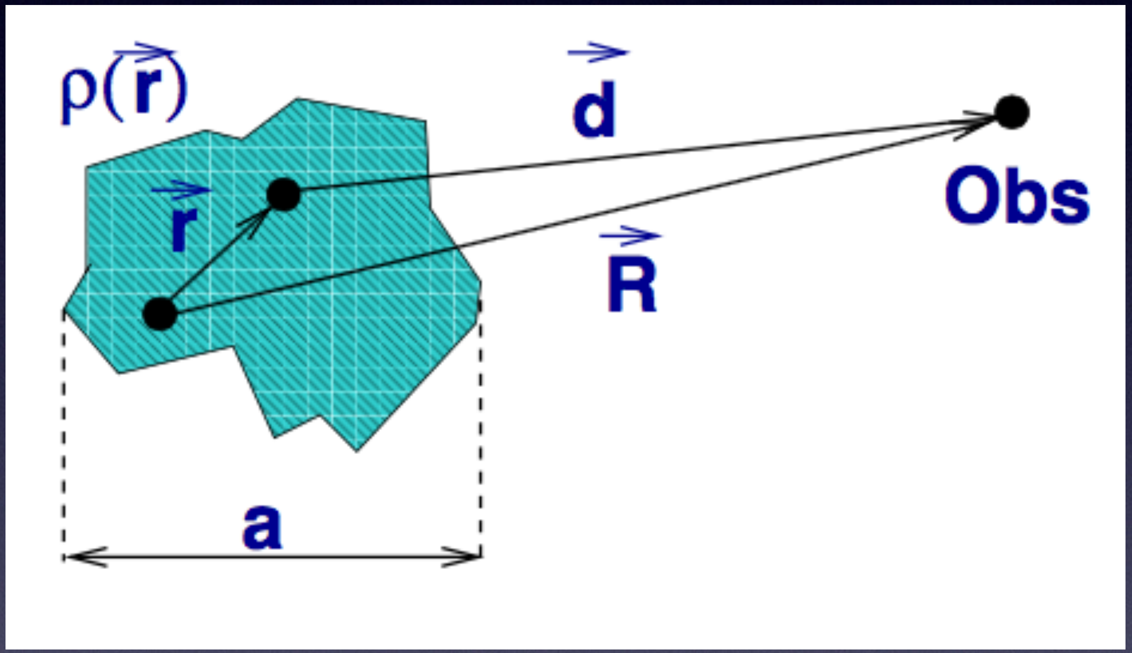
$$\Delta U = mgh\left(1 - \frac{h}{R} + \frac{h^2}{R^2} + \dots\right)$$

- h- Height above the Earth surface
- R- Radius of the Earth
- Theory is converge for h<R

Multipole Expansion

- Multipole expansion of electric potentials,

$$V = q\frac{1}{R} + p\frac{1}{R^2} + Q\frac{1}{R^3} + \ldots$$



- Sum converge for a << R

- Low energy probes are sensitive to only bulk properties.

EFT Algorithm

- Identify low energy scale and high energy scale in the theory. (Ration of scales will form expansion parameter)
- Identify the Symmetry of the theory
- Power Counting
- Write down Effective Lagrangian and interaction, consistent with the Symmetry
- Calculate loops and renormalize them

Nucleon-Nucleon Scattering at Low Energy

- Potential Model with strong interaction
- Asymptotic wave function and the cross section

$$\psi(\mathbf{r}) = \exp(\mathrm{i}\mathbf{k}_\mathrm{i} \cdot \mathbf{r}) + f(k,\theta)\frac{\exp(\mathrm{i}k_\mathrm{f} r)}{r}$$

$$\frac{d\sigma}{d\Omega} = |f(k,\theta)|^2$$

- T-Matrix (scattering amplitude)

$$T = -\frac{4\pi}{M}\frac{1}{k\cot\delta - ik}.$$

- Effective Range Expansion,

$$k\cot\delta(k) = -\frac{1}{a} + \frac{1}{2}r_0k^2 - Pr_0^3k^4 + \ldots.$$

  - a-scattering length
  - r0-effective range
  - P-shape parameter

- Leading ERE amplitude

$$T_{\mathrm{NN}}^{\mathrm{ERE}(0)} = \frac{4\pi a}{M} \frac{1}{1 + \mathrm{i}ak}$$

- Second order ERE amplitude

$$T_{\mathrm{NN}}^{\mathrm{ERE}(2)} = -\frac{4\pi}{M} \frac{1}{-\frac{1}{a} + \frac{1}{2} r_0 k^2 - \mathrm{i}k}.$$

- Analytical structure of T(k) must reproduce in Effective Field Theory

EFT for Nucleon Nucleon Scattering

- Build EFT to reproduce ERE Amplitude
- Low energy Degree of freedom - Nucleon
- Separation of Scale

$$k << M\pi$$

- Symmetry- Translational invariance, rotational invariance

- Non Relativistic-Nucleon Momentum $k<<M\pi$

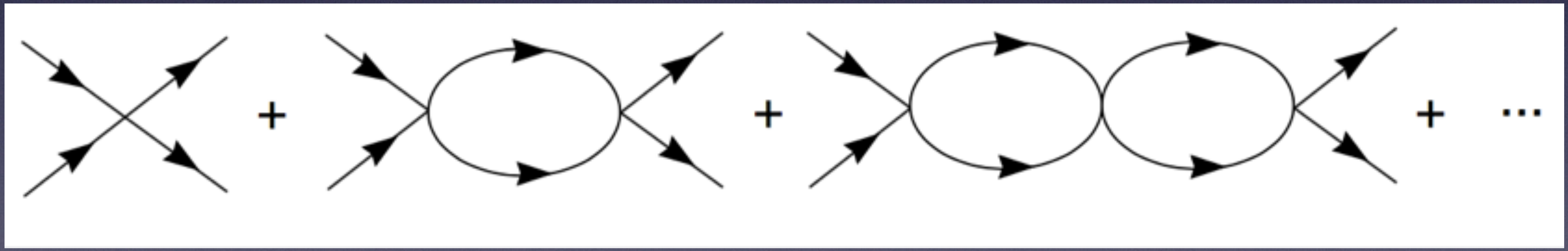
Effective Lagrangian

$$\mathcal{L} = N^\dagger \left(\mathrm{i}D_0 + \frac{\mathbf{D}^2}{2M} \right) N \; + \; \left(\frac{\mu}{2}\right)^{4-n} C_0 \left(N^{\mathrm{T}} P N\right)^\dagger \left(N^{\mathrm{T}} P N\right) + \ldots.$$

- P - projection operator for spin and isospin
- n - number of space -time dimension
- μ - arbitrary mass scale to regulate dimensions
- … - higher derivative operator for higher body interaction

Feynman Diagrams

- Sum Feynman diagrams which give the amplitude T to the desired order in k/Λ expansion.



- Summing Feynman diagrams, scattering amplitude from our effective Lagrangian,

$$T_{\mathrm{NN}} = [\, C_0 + C_0 I_0 C_0 + C_0 I_0 C_0 I_0 C_0 + \dots \,] \;=\; \frac{1}{1/C_0 - I_0}$$

I_0 - Momentum independent single loop integral

$$I_0 \;=\; -\mathrm{i}\left(\frac{\mu}{2}\right)^{4-n}\int \frac{\mathrm{d}^n q}{(2\pi)^n}\;\left(\frac{\mathrm{i}}{E + q_0 - \mathbf{q}^2/2M + \mathrm{i}\epsilon}\right)\left(\frac{\mathrm{i}}{-q_0 - \mathbf{q}^2/2M + \mathrm{i}\epsilon}\right)$$

  - here, Nucleon momentum, $k = \sqrt{ME}$

- This integral diverges and needs to be regularized and renormalized

MS Subtraction scheme

- Integral using MS subtraction scheme

$$I_0 = -M \left(-|\mathbf{k}|^2 - \mathrm{i}\epsilon\right)^{(n-3)/2} \left(\frac{\mu}{2}\right)^{4-n} \Gamma\left(\frac{3-n}{2}\right) (4\pi)^{(1-n)/2}$$

$$I_0^{\mathrm{MS}} = -\left(\frac{M}{4\pi}\right) \mathrm{i}|\mathbf{k}|,$$

Power Divergence Subtraction(PDS)

- new power counting
- this allows for fine tuning between coefficient C_0 and linear divergence.

$$I_0^{\mathrm{PDS}} = -\left(\frac{M}{4\pi}\right)(\mu + \mathrm{i}|\mathbf{k}|),$$

- μ is subtraction scale
- PDS isolate linear divergence from integral

- ERE amplitude,

$$T_{\rm NN}^{\rm ERE(2)} = -\frac{4\pi}{M}\frac{1}{-\frac{1}{a}+\frac{1}{2}r_0k^2-ik}.$$

- EFT amplitude,

$$T_{\rm NN} = [\, C_0 + C_0 I_0 C_0 + C_0 I_0 C_0 I_0 C_0 + \ldots\,] \;=\; \frac{1}{1/C_0 - I_0}$$

- Matching leading orders,

$$\frac{4\pi}{MC_0} = \frac{1}{a} - \mu.$$

- c_0 depends on both ‘a’ and meaning assigned to divergent part of ‘I_0’
- works with both ${}^3S_1, {}^1S_0$ channel

RADIATIVE NEUTRON CAPTURE ON CARBON-14

Motivation

- $^{14}C\ (n,\gamma)\ ^{15}C$ is important in Astrophysics
- Reaction is part of CNO cycle in helium burning stars
- Fundamental reaction in Big Bang Nucleosynthesis for production of nuclei with mass number > 14
- Theoretical method involved in radiative capture is important for future experimental efforts

Radiative Nutron capture on Carbon-14

- In EFT, short distance physics is not relevant at low energy
- Ground state of ^{15}C has angular momentum $J^{\pi} = (½)^{+}$
- Neutron separation energy 1.218 MeV
- Carbon-15 has a single Neutron halo bound to Carbon-145

- Final state ^{15}C : S-wave
- Incoming state n + ^{14}C : P-wave
- Transition from P-wave to S-wave
- Parity conservation - E1 (electric dipole transition) transition

Formalism

- EFT Lagrangian
- Feynman diagrams and scattering amplitude
- Cross section

EFT Lagrangian for final S Channel

$$\mathcal{L}_s = \phi_\alpha^\dagger[\Delta^{(0)} + i\partial_0 + \frac{\nabla^2}{2M}]\phi_\alpha + h^{(0)}[\phi_\alpha^\dagger(N_\alpha C) + \text{h. c.}]$$

- Auxiliary Field- C15 Field - ϕ_α
- Neutron Field - N_α
- Carbon-14 Field - C
- $M=M_n+M_c$

EFT Lagrangian for P Channel

$$\mathcal{L}_p = \chi_i^{\alpha,\eta\dagger}[\Delta^{(\eta)} + i\partial_0 + \frac{\nabla^2}{2M}]\chi_i^{\alpha,\eta} + \sqrt{3}h^{(\eta)}[\chi_i^{\alpha,\eta\dagger} P_{ik}^{\alpha\gamma,\eta} N_\gamma \left(\frac{\overrightarrow{\nabla}}{M_c} - \frac{\overleftarrow{\nabla}}{M_n}\right)_k C + \mathrm{h.c}],$$

- Here, projection operator,

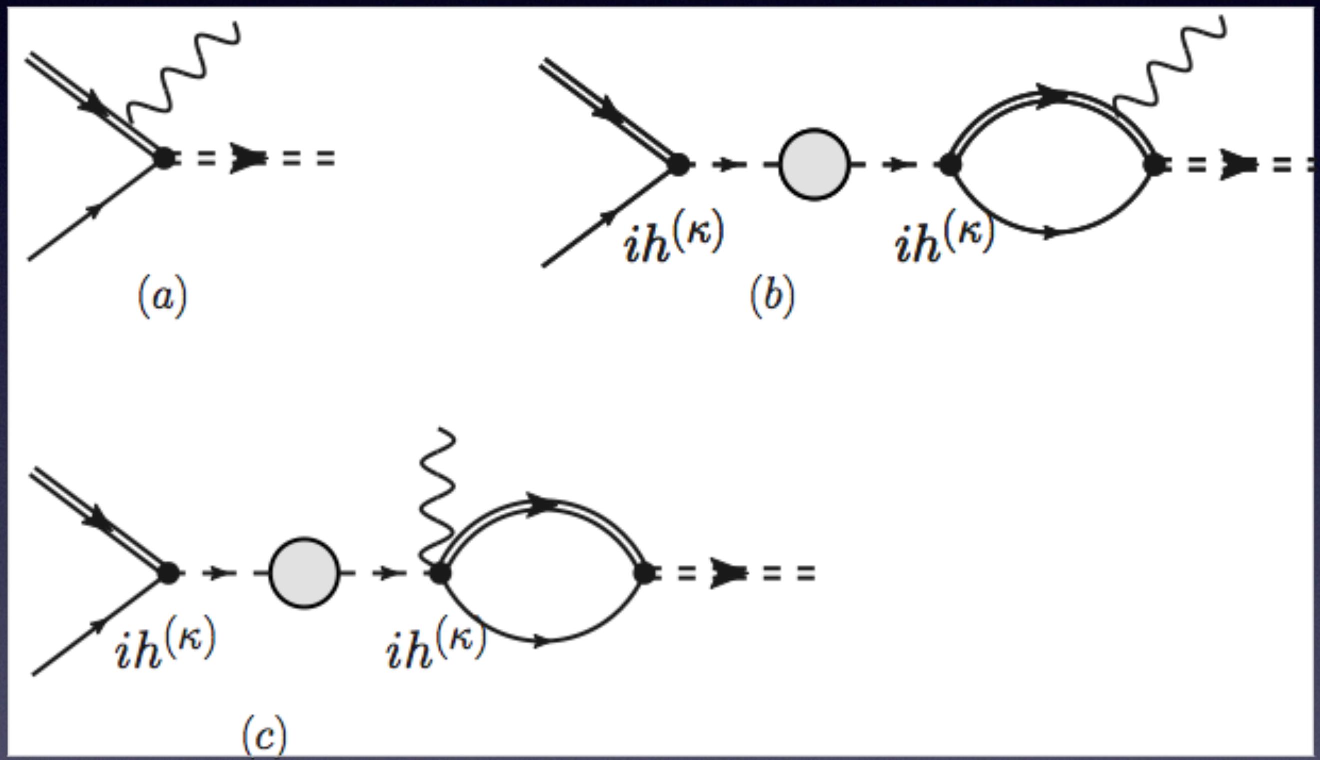
$$P_{ij}^{\alpha\beta,1} = \frac{1}{3}(\sigma_i\sigma_j)^{\alpha\beta} \rightarrow {}^2P_{\frac{1}{2}}\, channel$$

$$P_{ij}^{\alpha\beta,2} = \delta_{ij}\delta^{\alpha\beta} - \frac{1}{3}(\sigma_i\sigma_j)^{\alpha\beta} \rightarrow {}^2P_{\frac{3}{2}}\, channel$$

- Auxiliary field for p-channel - χ_i^{α}

Feynman Diagrams

- E1 Capture
- Single dashed line - dimer field - p-wave interaction - χ_i^α
- Double dashed line- dimer field - final state of C15 - ϕ_α



Scattering Amplitude

- $2p_{1/2}$ -Channel

$$|\mathcal{M}^{^2P_{1/2}}|^2 = \left|\frac{12eh_0\sqrt{Z_\phi}}{M_c}\right|^2 \frac{32M_nM_cMp^2}{9}\left|g^{2P_{1/2}}(p)\right|^2$$

$$g^{^2P_{1/2}}(p) = \frac{\mu}{p^2+\gamma^2} + \frac{6\pi\mu}{-1/a_1^{(1)} + r_1^{(1)}p^2/2 - ip^3}[\frac{\gamma}{4\pi} + \frac{ip^3-\gamma^3}{6\pi(p^2+\gamma^2)}],$$

- $2p_{3/2}$ -Channel

$$|\mathcal{M}^{^2P_{3/2}}|^2 = \left|\frac{12eh_0\sqrt{Z_\phi}}{M_c}\right|^2 \frac{16M_nM_cMp^2}{9}\left|g^{2P_{3/2}}(p)\right|^2(5-3\cos^2\theta),$$

$$g^{^2P_{3/2}}(p) = \frac{\mu}{p^2+\gamma^2} + \frac{6\pi\mu}{-1/a_1^{(2)} + r_1^{(2)}p^2/2 - ip^3}[\frac{\gamma}{4\pi} + \frac{ip^3-\gamma^3}{6\pi(p^2+\gamma^2)}]$$

Scattering Cross Section

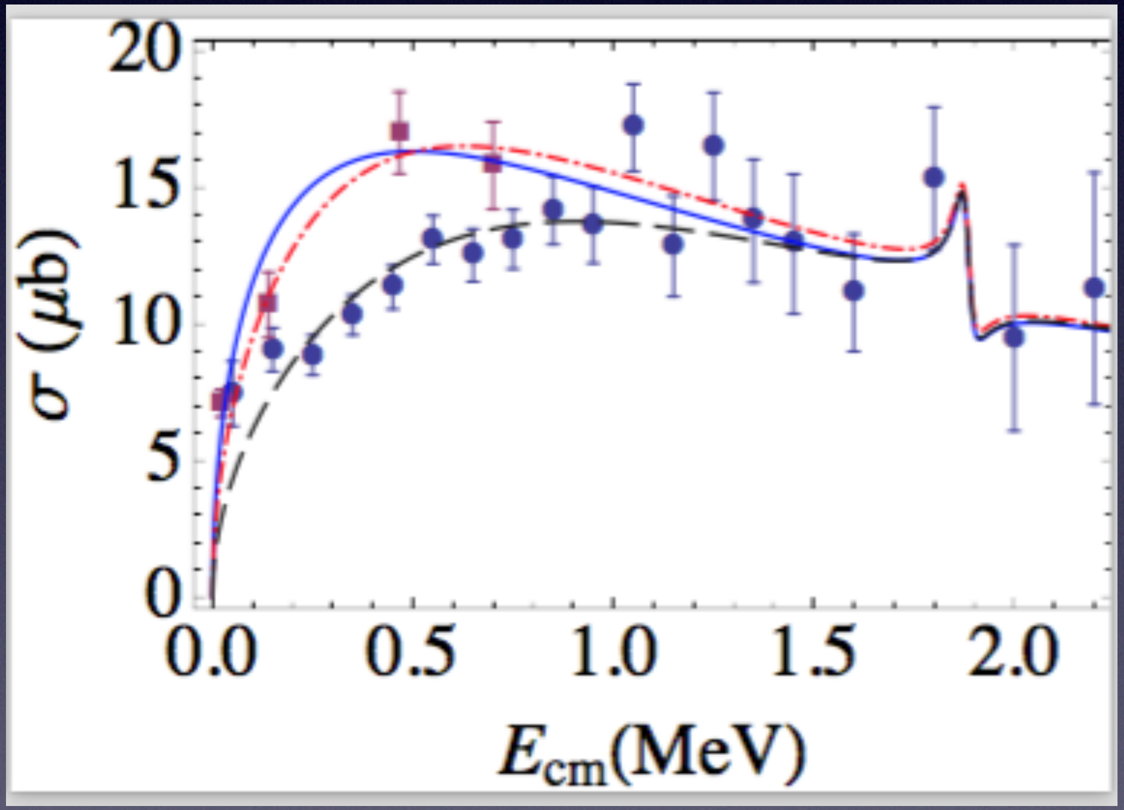
- Total cross section,

$$\sigma(p) = \frac{1}{2}\frac{64\pi\alpha}{M_c^2\mu^2}\frac{p\gamma(p^2+\gamma^2)}{1-\rho\gamma}[2|g^{^2P_{1/2}}(p)|^2 + 4|g^{^2P_{3/2}}(p)|^2]$$

- Depends on unknown EFT couplings that expressed in terms of ρ , a_1 and r_1

Results

- Parameterize EFT coupling
- ERE parameters ,
  a1 = $-n_1/(Q^3)$, r1 = $2n_2 Q$
- n_1, n_2 can estimate from coulomb dissociation and direct capture data



Form factors of single neutron halo system (C15)

- Halo nuclei play an important role in heavy element synthesis in nuclear astrophysics
- Clear separation of energy scales

  weakly bound valance nucleons and tightly bound core

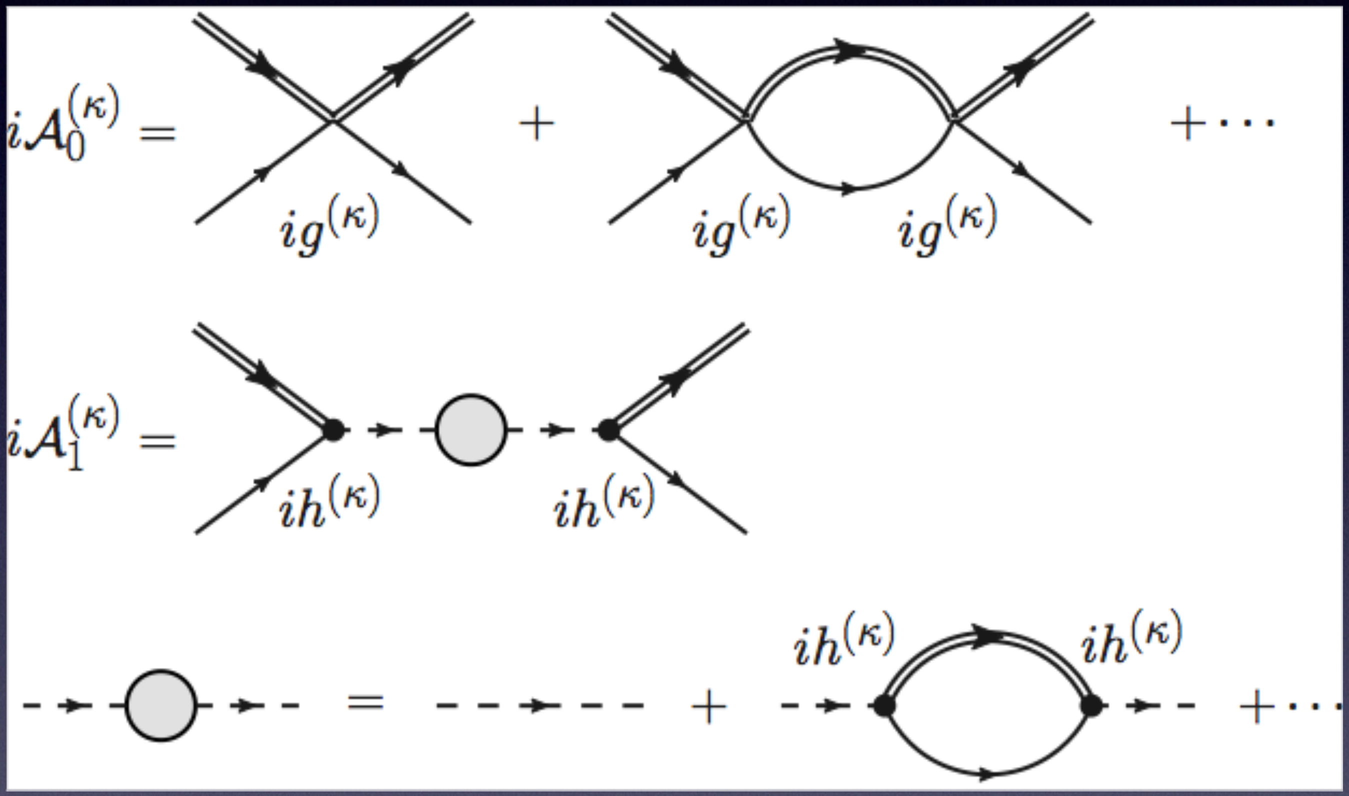
- Ideal to construct low energy EFT
- Form factor is important to determine internal properties like charge density, size, magnetic properties

- Probe internal structure of C15
- 15C - spin-1/2 single neutron halo
- 14C-core and single valence neutron
- Analyzed similar like electron scattering to Proton target

Elastic scattering and matching parameter

- EFT coupling relates with ERE parameter

$$\frac{2\pi\Delta}{\mu h^2} + \lambda = \gamma - \frac{1}{2}\rho\gamma^2,$$
$$-\frac{2\pi}{h^2\mu^2} = \rho$$

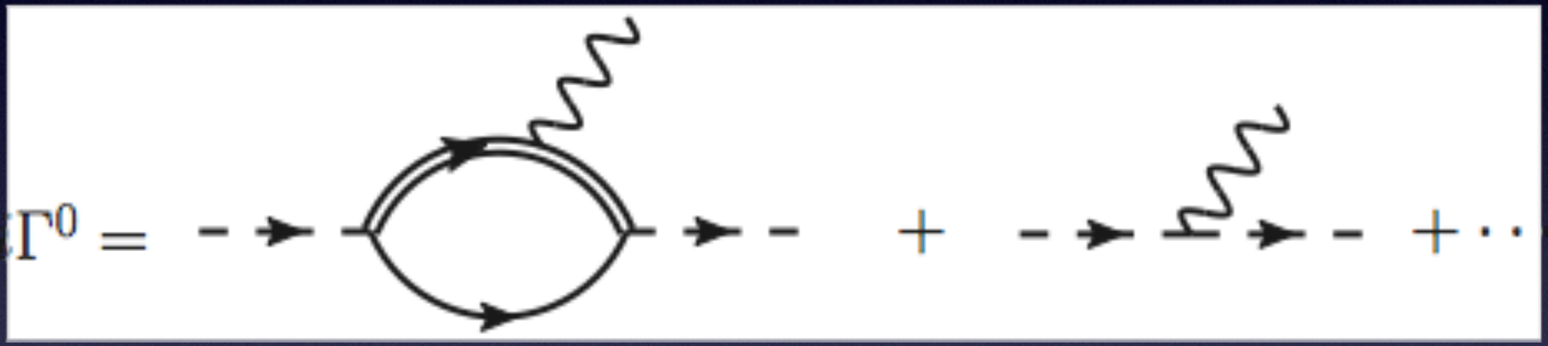


- Binding momentum $\gamma = \sqrt{2\mu B}$

- ρ - effective range

EFT Calculation of Γ^0 Photon

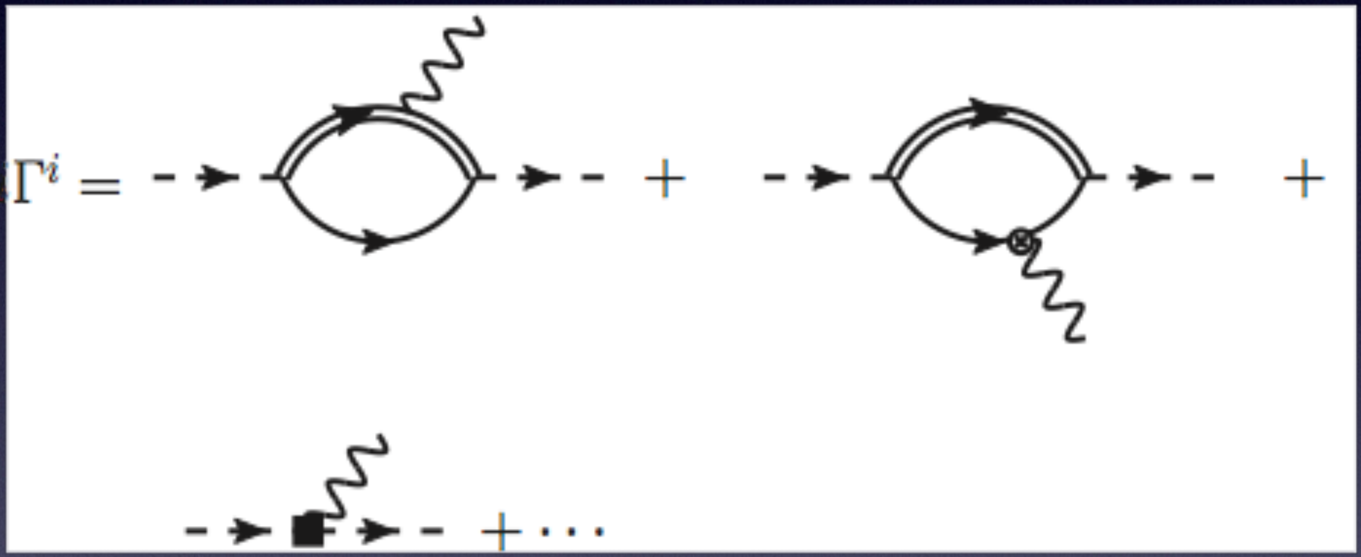
- Hadronic current - Electric Coupling



$$i\Gamma^0 = ieZ_c Z_\phi \bar{u}_\phi(p,a)[h^2 \frac{\mu M_c}{\pi|q|}\tan^{-1}(\frac{\mu|q|}{2M_c\gamma}) + 1]u_\phi(p',b)$$

EFT Calculation of Γ^i Photon

- Hadronic current - Magnetic coupling



$$i\Gamma^i = ieZ_cZ_\phi\bar{u}_\phi(p,a)\{\frac{p_i+p_i'}{2M}[h^2\frac{\mu M_c}{\pi|q|}\tan^{-1}(\frac{\mu|q|}{2M_c\gamma})+1]$$

$$+i\frac{\mu_N}{eZ_c}[h^2\kappa_n\frac{\mu M_n}{\pi|q|}\tan^{-1}(\frac{\mu|q|}{2M_n\gamma})+L_M]\epsilon^{ijk}\sigma_jq_k\}u_\phi(p',a')$$

Electric Form factor and charge radius

$$G_E(|q|^2) = Z_\phi[h^2 \frac{\mu M_c}{\pi |q|} \tan^{-1}(\frac{\mu |q|}{2M_c \gamma}) + 1] \approx 1 - \frac{\mu^2}{12 M_c^2 \gamma^2} \frac{1}{1-\rho\gamma} |q|^2 + \cdots ,$$

$$\langle r_E^2 \rangle = \frac{\mu^2}{2 M_c^2 \gamma^2} \frac{1}{1-\rho\gamma} + \langle r_c^2 \rangle ,$$

- determine charge radius entirely from binding energy

Magnetic Form Factor and Magnetic Moment

$$\frac{eZ_c}{2M}G_M(|q|^2) = \mu_N Z_\phi [\frac{h^2 g_n}{2}\frac{\mu M_n}{\pi|q|}\tan^{-1}(\frac{\mu|q|}{2M_n\gamma}) + L_M]$$

$$\approx (\kappa_n - L_M\rho\gamma)\mu_N\frac{1}{1-\rho\gamma} - \kappa_n\mu_N\frac{\mu^2}{12M_n^2\gamma^2}\frac{q^2}{1-\rho\gamma},$$

- Magnetic moment of halo nucleus,

$$\kappa_\phi = (\kappa_n - L_M\rho\gamma)\frac{1}{1-\rho\gamma} \approx k_n + (\kappa_n - L_M)\gamma\rho$$

Conclusion

- EFT describes experimental data consistently
- More accurate measurements needed near the resonance energy to verify the interference effect of resonance and non-resonance
- The form factor calculation of Carbon-15 will be useful for future form factor calculation of Beryllium 11(spin 1/2 halo)

Thank you